Lesson 7: Anti-Aliasing Filtering

ET 438b Sequential Control and Data Acquisition

Department of Technology

Learning Objectives

After this presentation you will be able to:

- Explain why anti-aliasing filters are used.
- Design a first order anti-aliasing filter using an OP AMP
- Design a 2nd order anti-aliasing filter using an OP AMP
- Verify the performance of an anti-aliasing filter using simulation software.

Anti-Aliasing Filters

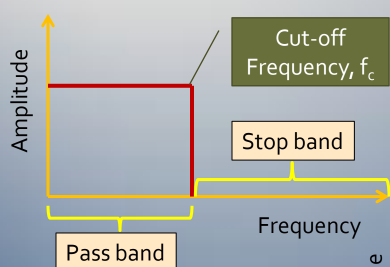
Dealing with Aliasing in practical systems

- Exact frequency components of a sampled signal are unknown.
  - Can not determine if signal component is alias or real
- Sampling rate limited by hardware selection.
  - Lab DAQ cards rate is 200 kHz-250 kHz.
  - $f_{nyquist}$ = 100 kHz – 125 kHz sets input frequency limits.
- Bandwidth limit input signals using anti-aliasing filters to eliminate frequencies above $f_{nyquist}$.

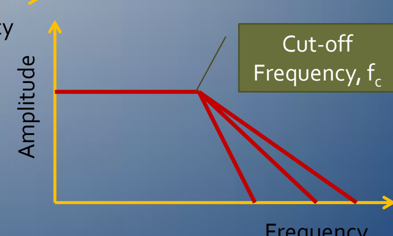
Anti-Aliasing Filters

Goal- reduce amplitude of all frequencies above $f_{nyquist}$ to zero level

Ideal low pass filter Bode plot



Set $f_c = f_{nyquist}$ for perfect signal elimination

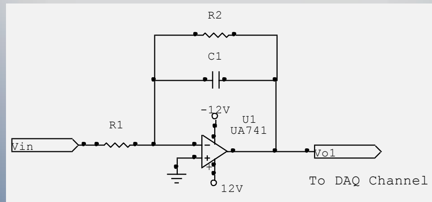


Practical (Butterworth) filters have sloping characteristics

1st Order: -20 dB/decade
2nd Order: -40 dB /decade
3rd Order: -60 db/decade

Anti-Aliasing Filters: OP AMP Circuits

First order Butterworth filter



Design formulas

$$A_v = \frac{-R2}{R1} \quad \text{Voltage gain in the pass band}$$

$$f_c = \frac{1}{2\pi \cdot R2 \cdot C1} \quad \text{Cut-off frequency (Hz)}$$

Design Procedure

1.) Determine the acceptable level of signal gain, A_v, at sampling frequency f_s

2.) Use the formula below to determine the value of f_c based of the designed signal gain

$$f_c = \frac{f_s}{2 \cdot \sqrt{\frac{1-A_v^2}{A_v^2}}} \quad \text{for } A_v < 1$$

3. Select a value of C1 from standard values and compute value of R2

Anti-Aliasing Filters: OP AMP Circuits

Design Procedure (continued)

4.) Set R1=R2 to give pass band gain of -1 (0 dB). Amplify signal after filtering to reduce noise and unwanted signal components. (e.g. 60 Hz)

<u>Design Example:</u> A data acquisition systems samples an analog signal at T_s=0.0004 seconds. Design a 1st order anti-aliasing filter that will reduce the voltage level of all signal above the Nyquist frequency to 0.4

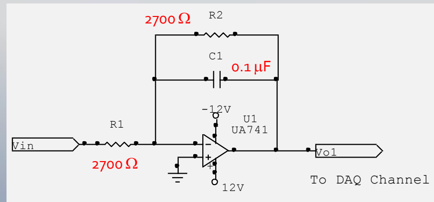
Solution: Determine the sampling frequency from T_s then find the $f_{nyquist}$

$$f_s = \frac{1}{T_s} = \frac{1}{0.0004\ \text{s}} = 2500\ \text{Hz} \qquad f_{nyquist} = \frac{f_s}{2} = \frac{2500\ \text{Hz}}{2} = 1250\ \text{Hz}$$

Find the value of f_c for the given level of A_v above $f_{nyquist}$

$$f_c = \frac{f_s}{2 \cdot \sqrt{\frac{1-A_v^2}{A_v^2}}} = \frac{2500\ \text{Hz}}{2 \cdot \sqrt{\frac{1-0.4^2}{0.4^2}}} = \frac{2500\ \text{Hz}}{2 \cdot \sqrt{5.25}} = 546\ \text{Hz}$$

Design Example (continued)



Select a capacitor value of 0.1 μF. Use larger values for lower f's to keep values of resistors in range of 1k to 820k

$$f_c = \frac{1}{2\pi \cdot R2 \cdot C1} \Rightarrow R2 = \frac{1}{2\pi \cdot C1 \cdot f_c}$$

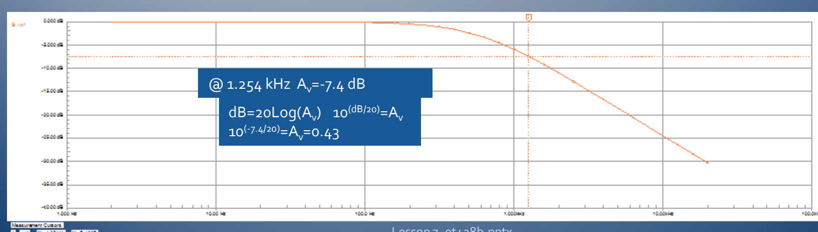
$$R2 = \frac{1}{2\pi \cdot 0.1\times10^{-6} \cdot 546} = 2916\,\Omega$$

Set R1=R2 to give gain of -1

Design complete. Check result with circuit simulation.

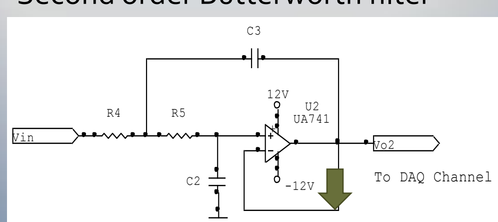
Select standard value of 2700 ohms and compute value of f_c

$$C1 = \frac{1}{2\pi \cdot 0.1\times10^{-6} \cdot 2700} = 590\text{ Hz}$$



Anti-Aliasing Filters- 2nd Order Filters

Second order Butterworth filter



Unity gain is fixed by negative feedback loop in this design.

Use same design procedure as 1st order filter but use following formula to find cut-off frequency.

$$f_c = \frac{f_s}{2 \cdot \sqrt[4]{\frac{1-A_v^2}{A_v^2}}} \quad \text{for } A_v < 1$$

Design formulas

$A_v = 1$ Voltage gain in the pass band

$$f_c = \frac{1}{2\pi \cdot \sqrt{R4 \cdot R5 \cdot C3 \cdot C2}} \quad \text{Cut-off frequency (Hz)}$$

With $R4 = R5$ and $C3 = 2 \cdot C2$

2nd order filter produces -40 dB/decade rolloff in stop band

Anti-Aliasing 2nd Order Filter Design

Design Example 2: Repeat the design of the 1st order filter example using a 2nd order filter

T_s=0.0004 seconds sampling. Design a 2nd order anti-aliasing filter that will reduce the voltage level of all signal above the Nyquist frequency to 0.4

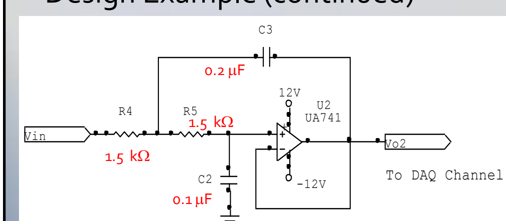
Same f_s and $f_{nyquist}$ as previous case

$$f_s = \frac{1}{T_s} = \frac{1}{0.0004\,\text{s}} = 2500\ \text{Hz} \qquad f_{nyquist} = \frac{f_s}{2} = \frac{2500\ \text{Hz}}{2} = 1250\ \text{Hz}$$

Find the value of f_c for the given level of A_v using 2nd order filter formula

$$f_c = \frac{f_s}{2\cdot\sqrt[4]{\frac{1-A_v^2}{A_v^2}}} = \frac{2500\ \text{Hz}}{2\cdot\sqrt[4]{\frac{1-0.4^2}{0.4^2}}} = \frac{2500\ \text{Hz}}{2\cdot\sqrt[4]{5.25}} = 826\ \text{Hz}$$

Design Example (continued)



Select value of C2 and compute C3. Let C2=0.1 µF so.....

C3= 2·C2=2·(0.1 µF)= 0.2 µF

Use design formulas to find value of R4=R5

$$f_c = \frac{1}{2\pi\cdot\sqrt{R4\cdot R5\cdot C3\cdot C2}} \quad \text{With } R4 = R5 \text{ and } C3 = 2\cdot C2$$

$$f_c = \frac{1}{2\pi\cdot\sqrt{R4^2\cdot 2\cdot C2^2}} = \frac{1}{2\pi\cdot\sqrt{2}\cdot R4\cdot C2}$$

Solve for R4 given C2 and f_c

$$f_c = \frac{1}{2\pi\cdot\sqrt{2}\cdot R4\cdot C2} \Rightarrow R4 = \frac{1}{2\pi\cdot\sqrt{2}\cdot C2\cdot f_c}$$

$$R4 = \frac{1}{2\pi\cdot\sqrt{2}\cdot C2\cdot f_c}$$

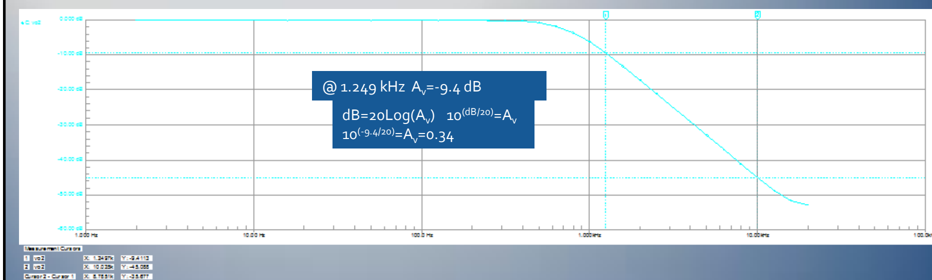
$$R4 = \frac{1}{2\pi\cdot\sqrt{2}\cdot 0.1\times10^{-6}\ \text{F}\cdot 826\ \text{Hz}}$$

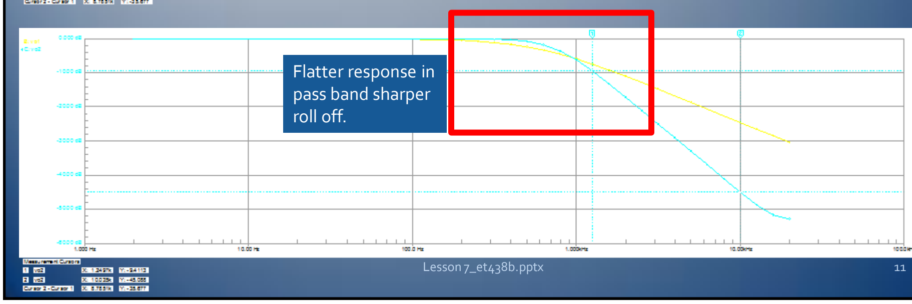
$$R4 = 1363\ \Omega = R5$$

Use standard value of 1.5k. Response very sensitive to R values

Design Example (continued)

Design complete. Check result with circuit simulation.





End Lesson 7: Anti-Aliasing Filtering

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